

**Четвертый список задач по курсу Дополнительные главы  
Математического Анализа**

Каждый должен научиться решать любую задачу из списка

10 ноября 2017

Простейшие задачи вариационного исчисления

1.  $\int_0^1 (1+t)\dot{x}^2 dt \rightarrow \text{extr}, x(0) = 0, x(1) = 1;$
2.  $\int_1^2 t^2 \dot{x}^2 dt \rightarrow \text{extr}, x(1) = 3, x(2) = 1;$
3.  $\int_0^1 x^2 \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 1, x(1) = \sqrt{2};$
4.  $\int_0^1 e^x \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 0, x(1) = \ln 4;$
5.  $\int_0^1 (\dot{x}^2 + x\dot{x} + 12tx) dt \rightarrow \text{extr}, x(0) = 0, x(1) = 0;$
6.  $\int_{-1}^1 (\dot{x}^2 + x^2) dt \rightarrow \text{extr}, x(-1) = 1, x(1) = 1;$
7.  $\int_{-1}^1 (\dot{x}^2 + 4x^2) dt \rightarrow \text{extr}, x(-1) = -1, x(1) = 1;$
8.  $\int_0^1 (\dot{x}^2 + x^2 + 2x) dt \rightarrow \text{extr}, x(0) = 0, x(1) = 0;$
9.  $\int_0^1 (4x \sin t - x^2 - \dot{x}^2) dt \rightarrow \text{extr}, x(0) = 0, x(1) = 0;$
10.  $\int_0^1 (\dot{x}^2 + x^2 + tx) dt \rightarrow \text{extr}, x(0) = 0, x(1) = 0;$

Простейшие задачи с фиксированными параметрами

1.  $\int_0^{T_0} \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
2.  $\int_0^{T_0} (\dot{x}^2 - x^2) dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
3.  $\int_0^{T_0} \dot{x}^3 dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
4.  $\int_0^{T_0} (\dot{x}^3 - \dot{x}^2) dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
5.  $\int_0^{T_0} (\dot{x}^3 + x^2) dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
6.  $\int_0^{T_0} (\dot{x}^2 + x^2 - 4x \sin t) dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$
7.  $\int_0^{T_0} (\dot{x}^2 + x^2 + 4x \operatorname{sh} t) dt \rightarrow \text{extr}, x(0) = 0, x(T_0) = \xi;$

Задачи Больца

1.  $\int_0^1 (\dot{x}^2 + x^2) dt - 2x(1) \operatorname{sh} 1 \rightarrow \text{extr};$
2.  $\int_0^{\pi/2} (\dot{x}^2 - x^2) dt + x^2(0) - x^2(\pi/2) + 4x(\pi/2) \rightarrow \text{extr};$
3.  $\int_1^e 2\dot{x}(t\dot{x} + x) dt + 3x^2(1) - x^2(e) - 4x(e) \rightarrow \text{extr}.$

Задачи общего вида

1.  $\int_0^1 \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 1;$

2.  $\int_0^T \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 0, T + x(T) + 1 = 0;$
3.  $\int_0^T \dot{x}^2 dt \rightarrow \text{extr}, x(0) = 0, (T - 1)x^2(T) + 2 = 0;$
4.  $\int_0^1 (\dot{x}^2 + x) dt \rightarrow \text{extr}, x(1) = 0;$
5.  $\int_0^T (\dot{x}^2 + x) dt \rightarrow \text{extr}, x(0) = 1;$
6.  $\int_0^T (\dot{x}^2 + x) dt \rightarrow \text{extr}, x(T) = T;$
7.  $\int_0^T (\dot{x}^2 + x) dt \rightarrow \text{extr}, x(0) = 0, x(T) = T;$
8.  $\int_0^T (\dot{x}^2 + x + 2) dt \rightarrow \text{extr}, x(0) = 0;$
9.  $\int_0^{\pi/4} (\dot{x}^2 - x^2) dt \rightarrow \text{extr}, x(0) = 1;$
10.  $\int_0^1 (\dot{x}^2 + x^2) dt \rightarrow \text{extr}, x(0) = 1;$
11.  $\int_0^T (\dot{x}^2 + x^2) dt \rightarrow \text{extr}, x(0) = 0, x(T) = 1.$